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Vibrational modes in plasma crystals due to nonlinear temperature distribution in gas discharge plasmas

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It is shown that a nonlinear temperature distribution in gas discharge plasma leads to a specific low-frequency mode of a quasi-two-dimensional plasma crystal. Linear dispersion characteristics of the mode are obtained. The characteristics of the mode can depend strongly on the temperature gradients and therefore can be effectively controlled by the experimental conditions.

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Since the prediction of possibility of formation of Coulomb lattices involving highly charged dust microparticles [1], interest in theoretical and experimental studies of these strongly coupled structures has grown tremendously [2,3]. The plasma crystal formation is usually observed in the sheath region where there is a balance between the gravitational and electrostatic forces [2–7]. This results in a few horizontal particle layers levitating above the negatively charged electrode. Such plasma crystal structures can support longitudinal and transverse vibrational modes, which are extensively studied in past years [4–15]. Note that properties of these modes strongly depend on the ambient plasma conditions because of the open character of complex plasma systems [14,16].

There are recent experimental observations of the formation of complex plasma structures suspended in a gas discharge plasma by thermophoresis [17,18]. Contrary to the particle levitation in the sheath electric field where strong inhomogeneities of the plasma and ion flows take place leading to specific collective effects such as generation of the wake fields [19–21], the thermoforetic force can lift up the microparticles outside the sheath region into the central part of the discharge chamber. This has opened new opportunities for the detailed observations of the particle behavior at the kinetic level in the plasma bulk and in particular investigate a void structure [17,22,23]. However, considering dynamics of the charged particulates in a gas discharging plasma, it was tacitly agreed that the temperature is a linear function of the electrode distance. At the same time, recent modeling of the heat flow processes in gas discharge plasmas in a consistent way exhibits a nonlinear distribution of the gas temperature as a function of the distance between electrodes [24]. Such profile demonstrates a strong dependence on the boundary conditions, but always reveals local maxima near the electrodes and a minimum in a chamber center, as sketched on Fig. 1. Note that, in general, the minimum in the center of the chamber appears only in the z direction; in the direction parallel to the electrodes this point corresponds to the local maximum and, thus, the center of chamber exhibits the saddle point in the three-dimensional temperature distribution [24].

Here, we investigate the particle vibrational modes in the presence of the nonlinear temperature profile. We show that the nonlinear temperature distribution of the neutral gas results in the specific mode of vertical oscillations of dust grains, which can be stable or unstable depending on the curvature of the temperature profile at the levitation latitudes, thus, providing a tool for control of complex plasma crystal experiments and determination of the experimental parameters.

Consider the standard model of a one-dimensional horizontal chain [9,12], where particulates have the charge Q, mass M, and are separated by the same distance Δ, see Fig. 2. The electrostatic potential of each particle is assumed to be the screened Debye-Hückel potential

$$\Phi=\frac{Q}{r}\exp\left(-\frac{r}{\lambda_D}\right), \tag{1}$$

where λ_D is the screening length of a plasma. Usually the particle separation in dust-plasma crystals Δ exceeds the screening length λ_D, viz. $\Delta/\lambda_D \sim 1.5-2.5$, so that we take only the nearest neighbor particle interactions into account.

The total force acting on a dust particle in a vertical direction is in general a combination of three forces: gravitational force Mg, the electrostatic force $F_e=QE(z)$, and the thermoforetic force F_T, the latter arising due to the temperature gradient of a neutral gas ∇T_n. The force F_T acting on a spherical particle in a monoatomic gas at a low pressure, where the mean free path is much larger than the particle radius, is given by [18,25]

$$F_T=-3.33\kappa\frac{a^2}{\sigma}\nabla T_n. \tag{2}$$

Here, a is the particulate radius, κ is the Boltzmann constant, and σ denotes the gas kinetic cross section for the atomic scattering, which for noble gases is $\approx(15\text{–}67)\times10^{-20}$ m^2 [26]. Furthermore, since for vertical vibrations in the labora-

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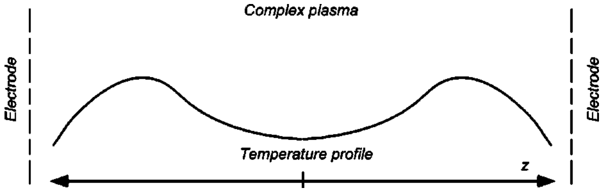

FIG. 1. Typical temperature distribution inside the chamber.

tory plasmas the vertical temperature gradients are the most important [17,18], we assume $\nabla T_n \to \partial T_n/\partial z$.

The balance of these forces in the linear approximation with respect to small perturbations $\delta z \ll \Delta$ of the equilibrium at $z=0$ gives the equation for vertical oscillations

$$\frac{d^2\delta z_n}{dt^2}=\frac{Q^2}{M\Delta^3}\exp\left(-\frac{\Delta}{\lambda_D}\right)\left(1+\frac{\Delta}{\lambda_D}\right)\times(2\,\delta z_n-\delta z_{n-1}-\delta z_{n+1})-g+\frac{F_e}{M}-A\frac{\partial T_n}{\partial z},\tag{3}$$

where the factor A is defined by

$$A=3.33\kappa\frac{a^2}{\sigma M}\tag{4}$$

and δz_n is the vertical displacement of the nth particle. The equilibrium balance of the main forces in Eq. (3) yields $gM=QE(0)-A(\partial T_n/\partial z)_{z=0}$, so that the last three terms in Eq. (3) are expanded at the particle equilibrium to order δz_n as well,

$$-g+\frac{1}{M}F_e-A\frac{\partial T_n}{\partial z}=-\beta\,\delta z_n.\tag{5}$$

To find an expression for β, we have to make some further assumptions regarding the main forces involved in Eq. (5). For the case, when thermal gradients in a discharge plasma are small and the electric force due to the sheath electric field dominates over the thermoforetic force, $F_e \gg F_T$, the last term in the left-hand side of Eq. (5) can be neglected and we recover the well-known frequency of the vertical oscillations in the sheath region [12]

$$\beta\to\Omega_E^2=-\frac{Q}{M}\left(\frac{\partial E(z)}{\partial z}\right)_{z=0}.\tag{6}$$

The situation is different when the thermal gradients become significant and the force balance is achieved mainly due to the thermoforetic force $F_T \gg F_e$. Then the frequency of the vertical vibrations is mainly determined by the nonlinear temperature distribution between the electrodes, and is given by

$$\beta\to\Omega_T^2=A\left(\frac{\partial^2 T_n}{\partial z^2}\right)_{z=0}.\tag{7}$$

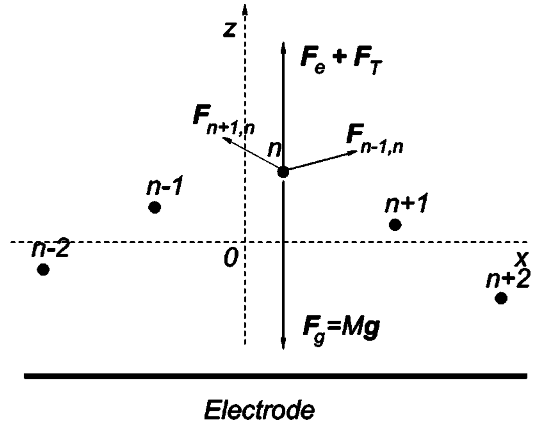

FIG. 2. The one-dimensional chain and vertical oscillations of particles.

Note that this frequency does not depend on particle charges and can easily be controlled externally by changing the laboratory conditions.

Although, in general, the particles oscillate in the vertical as well as in the horizontal directions, in the linear approximation the longitudinal and transverse motions are decoupled. Assuming that δz_n varies as $\sim\exp(-i\omega t+ikn\Delta)$, Eq. (3) gives the dispersion relation

$$\omega^2=A\left(\frac{\partial^2 T_n}{\partial z^2}\right)_{z=0}-4\frac{Q^2}{M\Delta^3}\exp\left(-\frac{\Delta}{\lambda_D}\right)\times\left(1+\frac{\Delta}{\lambda_D}\right)\sin^2\frac{k\Delta}{2}.\tag{8}$$

If $(\partial^2 T_n/\partial z^2)_{z=0}>0$, then Eq. (8) is similar to the dispersion dependence for vertical vibrations in the sheath region [12]: the mode is characterized by an optical-mode-like inverse dispersion (the maximum frequency is achieved at $k=0$ and then the frequency decreases with the growing wave number k). In another limiting case, when the force balance occurs in the region where $(\partial^2 T_n/\partial z^2)_{z=0}<0$, the vertical mode is always unstable. Thus, we expect that stable vertical vibrations are most likely to occur only in the central region of the discharge plasma chamber where $(\partial^2 T_n/\partial z^2)_{z=0}>0$.

To obtain a numerical estimate of the frequency Ω_T, we consider typical plasma parameters of laboratory experiments on application a temperature gradient for gravity compensation [17,18]. Measurements were done in a rf-excited plasma for melamine resin particles (density 1510 kg/m^3) with a radius $a=1.7$ μm. Taking the gas kinetic cross section for argon as $\sigma=42\times10^{-20}$ m^2 [26], we have $A=10^{-2}$ kg m^2/s^2 K. Hence, we need only to specify a value of $\partial^2 T_n/\partial z^2$ to calculate the frequency of vertical oscillations (7). However, up to now the temperature profiles in complex plasma experiments have not been studied enough and there are no direct measurements of $T_n(z)$. Even a model presentation of the temperature distribution is strongly dependent on the wall conditions [24]. A rough estimate of $(\partial^2 T_n/\partial z^2)_{z=0}$ can be obtained using an inequality $(\partial^2 T_n/\partial z^2)_{z=0}\Delta \ll (\partial T_n/dz)_{z=0}$. With $\Delta\sim100$ μm and $\partial T_n/dz\sim10^3$ K/m (the particle equilibrium was reached due to an averaged temperature gradient of 1170 K/m [17,18]), this implies $(\partial^2 T_n/\partial z^2)_{z=0} \sim 10^5 - 10^6$ K/m^2 and then $\Omega_T \sim 30$–100 s^{-1}. Thus, we expect that the vertical modes arising due to the thermoforetic force can be characterized by the frequency of the same order as the vibrations due to the sheath electric field force Ω_E, which are successfully used for complex plasma diagnostics [27].

To summarize, we have demonstrated that vertical oscillations of a one-dimensional chain of dust grains supported by thermoforetic force give rise to a specific low-frequency mode which is characterized by inverse optic-mode-like dispersion when the wavelength far exceeds the intergrain distance. These oscillations can provide a tool for determining the temperature profile (e.g., using particles of different sizes, levitated at different altitudes). The characteristics of the mode in the regime when they are determined by the temperature gradients can be effectively controlled by the change of experimental conditions. Thus, thermal properties of complex plasma crystals qualitatively differ from those of solid state ionic crystals; indeed, they appear to be related more to the corresponding plasma and neutral gas state than to the particle excitations. This opens new possibilities for investigation of particle behavior at the kinetic level as well as for stimulating phase transitions in the system, and for studies of self-organized structures (e.g., voids) appearing in the experiments, including those under microgravity conditions.

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